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Mixed-mode fracture of brittle cellular materials

J. S. HUANG*, J. Y. LIN
Department of Civil Engineering, National Cheng Kung University, Tainan, 70101 Taiwan

Dimensional argument analysis and near-tip singular in-plane shear stress of a continuum model have been employed to derive the expression for mode II fracture toughness of brittle cellular materials. It was found that both mode I and II fracture toughnesses have the same dependence on cell size, relative density and modulus of rupture of solid cell walls, except a microstructure coefficient included in their expressions. In addition, the linear superposition principle was applied to calculate the bending moment exerted at the first unbroken cell wall for brittle cellular materials under a combined loading of uniform tensile and in-plane shear stresses. The resulting mixed-mode fracture criterion was compared to existing experimental data in PVC foams; agreement was found to be good.

1. Introduction

There is a growing interest in the use of sandwich panels with ceramic cellular cores as load-bearing components in lightweight structures. For example, sandwich panels with a cementitious foam core and gypsum faces are typically used in building. Ceramic cellular materials have excellent thermal insulation and fire resistance but are brittle. Pre-existing cracks in brittle cellular materials, resulting from manufacturing or machining, might cause catastrophic failure at a tensile stress much less than the yielding strength. Under some circumstances, the crack surface may not be perpendicular to the imposed in-plane shear stress which is the primary loading of core materials in sandwich panels, producing a mode II or mixed-mode fracture. The mixed-mode fracture criteria for solid materials are invalid for cellular materials because failure mechanisms are different. Therefore, mixed-mode fracture of brittle cellular materials needs to be fully exploited to understand crack propagation in cellular materials and to ensure structural integrity of sandwich panels.

Gibson and Ashby [1] proposed a bending model of cell walls to analyse mechanical properties of cellular materials. They found that mechanical properties of cellular materials are related to cell geometry and material properties of solid cell-wall materials. Fowlkes [2] measured mode I fracture toughness of a rigid polyurethane foam from various types of specimens to verify the applicability of linear elastic fracture mechanics to the fracture of foams. McIntyre and Anderton [3] confirmed the dependence of relative density of rigid polyurethane foams on their mode I fracture toughness: K^*_{IC} increases with increasing relative density. Bulk and microscopic models for the mechanical behaviour of cellular glass were attemped by Zwissler and Adams [4]. It was found that fracture strength, tensile elastic modulus and fracture toughness increase linearly with density. The influence of anisotropy on the fracture toughness of woods, which have a similar microstructure to honeycombs, was studied by Ashby *et al.* [5]. The fracture toughness of cellular materials is directionally dependent if they are not isotropic. The mode I, II and mixed-mode fracture of PVC foams were investigated by Zenkert and Backlund [6, 7]. They found that K^*_{IIC} is slightly larger than K^*_{IC}.

Maiti *et al.* [8] utilized the bending model of cell walls in cooperation with the near-tip singular tensile stress of a continuum model to derive the expression for mode I fracture toughness of cellular materials. Results indicated that K^*_{IC} of cellular materials depends on cell size, relative density and modulus of rupture of solid cell walls. In their modelling, the modulus of rupture of solid cell walls was assumed to be constant. In practice, the modulus of rupture of a brittle cell wall is mainly controlled by its volume, giving a cell-size effect. Cell-size effects on fracture strength of foamed glass [9] and on mode I fracture toughness of reticulated vitreous carbon foam [10], were observed. Huang and Gibson [11, 12] studied the variable strength of solid cell walls using Weibull statistic analysis. It was noted that the Weibull modulus of solid cell walls plays an important role in determining optimum cell size to obtain a higher value of K^*_{IC}.

The present work was aimed at deriving the expression for mode II fracture toughness and a mixed-mode fracture criterion because they are essential for the analysis of crack propagation in brittle cellular materials. The bending model of cell walls, dimensional argument, as well as the near-tip singular stress field of linear elastic fracture mechanics, were employed to analyse the mode II and mixed-mode fracture for

* Author to whom all correspondence should be addressed.

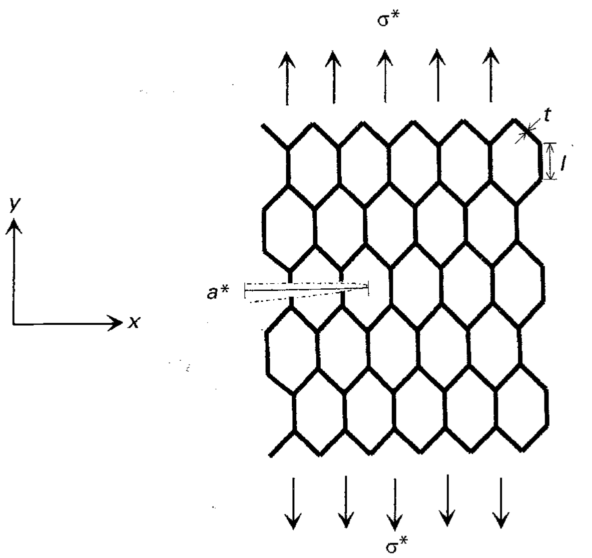

Figure 1 An infinite honeycomb plate with a central crack under a uniform tensile stress.

honeycombs and foams. The mixed-mode fracture criterion from the theoretical modelling has been compared with existing experimental data in PVC foams [7].

2. Honeycombs

A typical honeycomb with a central crack, a^*, cell length, l, and cell wall thickness, t, is under a remote uniform tensile stress, σ^*, as shown in Fig. 1. The expression for mode I fracture toughness is found to be [8]

$$K_{\mathrm{IC}}^* = C_1 \sigma_{\mathrm{fs}} (\pi l)^{1/2} \left(\frac{\rho^*}{\rho_s}\right)^2 \tag{1}$$

where C_1 is a microstructure coefficient and was numerically found to be 0.18 by Huang and Chiang [13], and σ_{fs} is the modulus of rupture of solid cell walls. ρ^* and ρ_s are the densities of honeycombs and the solid material from which they are made, respectively.

2.1. Mode II fracture of brittle honeycombs

Fig. 2 illustrates a honeycomb plate with a central crack, a^*. It is assumed that the crack length is much larger than the cell size of honeycombs. A remote uniform in-plane shear stress, τ^*, is imposed on the outermost layer of the honeycomb plate, generating a singular near-tip stress field in linear elastic fracture mechanics [14]. The resulting in-plane shear stress, τ_{xy}, along the crack surface is

$$\tau_{xy} = \frac{K_{\mathrm{II}}^*}{(2\pi r)^{1/2}} = \frac{\tau^* (\pi a^*)^{1/2}}{(2\pi r)^{1/2}} \tag{2}$$

The distance, r is measured from the crack tip located at the centre of the first unbroken cell, as shown in Fig. 2. K_{II}^* is the mode II stress intensity factor and is proportional to $\tau^*(\pi a^*)^{1/2}$. The expression for mode II fracture toughness of honeycombs can be derived by using dimensional argument analysis in conjunction

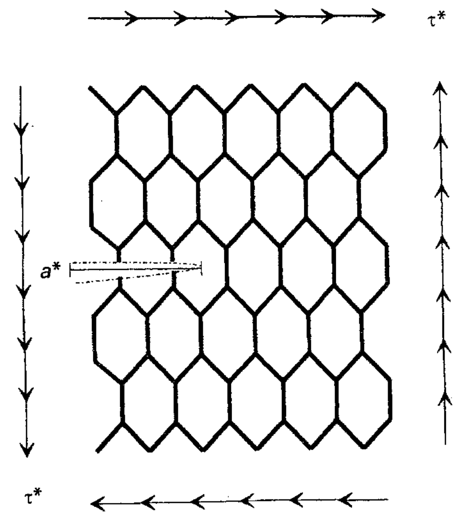

Figure 2 An infinite honeycomb plate with a central crack under a uniform in-plane shear stress.

with Equation 2. That is, the integration of the resulting in-plane shear stress over a distance of one cell size gives the total shear force carried by the first unbroken cell wall

$$V \propto \int_0^{2l\cos\theta} \tau_{xy} b \mathrm{d}r$$
$$\propto \tau^* b (a^* l)^{1/2} \tag{3}$$

The bending moment exerted at the first unbroken cell wall is proportional to Vl

$$M \propto \tau^* b l (a^* l)^{1/2} \tag{4}$$

The critical skin stress of the first unbroken cell wall can be calculated from the elementary mechanics of materials

$$\sigma_c \propto \frac{M}{bt^2}$$
$$\propto \tau^* \left(\frac{l}{t}\right)^2 \left(\frac{a^*}{l}\right)^{1/2} \tag{5}$$

The crack advances when the critical skin stress reaches the modulus of rupture of solid cell walls. At the moment, the imposed uniform shear stress has a maximum value called the fracture strength, τ_f^*

$$\tau_f^* \propto \sigma_{\mathrm{fs}} \left(\frac{t}{l}\right)^2 \left(\frac{l}{a^*}\right)^{1/2} \tag{6}$$

Once fracture strength and crack length are known, the mode II fracture toughness of the honeycomb can be calculated

$$K_{\mathrm{IIC}}^* = \tau_f^* (\pi a^*)^{1/2}$$
$$\propto \sigma_{\mathrm{fs}} \left(\frac{t}{l}\right)^2 (\pi l)^{1/2} \tag{7}$$

The relative density of the honeycomb, ρ^*/ρ_s, is proportional to t/l. As a result, the expression for mode II fracture toughness of honeycombs can be written as

$$K_{\mathrm{IIC}}^* = C_2 \sigma_{\mathrm{fs}} (\pi l)^{1/2} \left(\frac{\rho^*}{\rho_s}\right)^2 \tag{8}$$

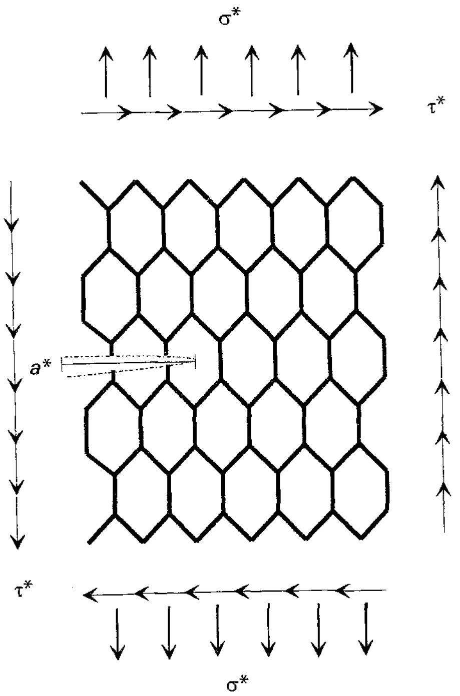

Figure 3 An infinite honeycomb plate with a central crack under a combined loading of uniform tensile and in-plane shear stresses.

where C_2 is a microstructure coefficient and must be determined empirically or numerically.

2.2. Mixed-mode fracture of brittle honeycombs

An infinite honeycomb plate under a combined loading of uniform tensile and in-plane shear stresses is shown in Fig. 3. It is assumed that brittle honeycombs are linear elastic up to fracture, resulting in the applicability of linear superposition principle in elasticity. The bending moment acting at the first unbroken cell wall for the honeycomb subject to a single tensile stress and a single in-plane shear stress can be calculated, respectively. Linear superposition of the two resulting bending moments gives

$$M = d_1\sigma^* bl(a^* l)^{1/2} + d_2\tau^* bl(a^* l)^{1/2} \quad (9)$$

where d_1 and d_2 are constants depending on cell geometry of honeycombs. Because bending moment dominates cell-wall deformation in cellular materials, the critical skin stress of the first unbroken cell wall is hence found to be

$$\sigma_c \propto \frac{M}{bt^2} \propto \frac{(d_1\sigma^* + d_2\tau^*)\, l(a^* l)^{1/2}}{t^2} \quad (10)$$

When the critical skin stress exceeds the modulus of rupture of solid cell walls, mixed-mode fracture will occur. Therefore, the maximum combined loading of uniform tensile and in-plane shear stresses $(d_1\sigma^* + d_2\tau^*)_f$, at which crack propagates, is:

$$(d_1\sigma^* + d_2\tau^*)_f(\pi a^*)^{1/2} \propto \sigma_{fs}\left(\frac{t}{l}\right)^2 (\pi l)^{1/2} \quad (11)$$

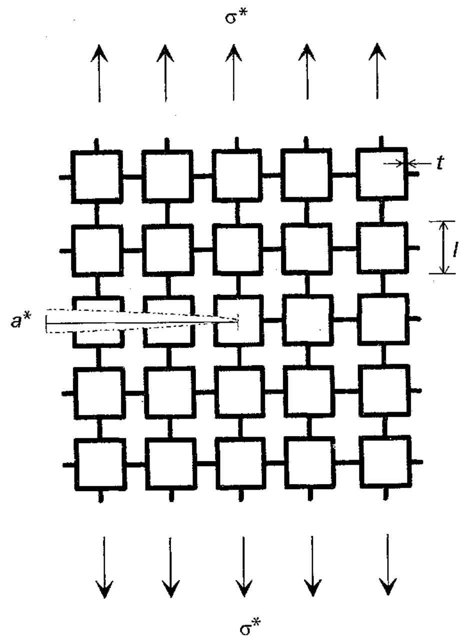

Figure 4 An idealized model of foam plate with a central crack under a uniform tensile stress.

The left-hand side of the above expression represents a linear combination of mode I and II stress intensity factors. Thus, the mixed-mode fracture criterion for honeycombs can be expressed as

$$d_1 K_I^* + d_2 K_{II}^* = d_3\sigma_{fs}(\pi l)^{1/2}\left(\frac{\rho^*}{\rho_s}\right)^2 \quad (12)$$

Here d_3 is another microstructure coefficient of honeycombs. For a specific honeycomb, the right-hand side in Equation 12 is constant, regardless of the magnitude of imposed tensile and in-plane shear stresses. Dividing both sides of Equation 12 by $\sigma_{fs}(\pi l)^{1/2}(\rho^*/\rho_s)^2$ and then employing the relationships between K_{IC}^*, K_{IIC}^* and $\sigma_{fs}(\pi l)^{1/2}(\rho^*/\rho_s)^2$ (Equations 1 and 8) to rearrange the mixed-mode fracture criterion as

$$\frac{d_1 K_I^*}{K_{IC}^*/C_1} + \frac{d_2 K_{II}^*}{K_{IIC}^*/C_2} = d_3 \quad (13)$$

The above mixed-mode fracture criterion must be applicable for the two special cases, $K_I^* = K_{IC}^*$ for mode I fracture and $K_{II}^* = K_{IIC}^*$ for mode II fracture. Two relationships are obtained, $d_1 C_1 = d_3$ and $d_2 C_2 = d_3$. The mixed-mode fracture criterion for brittle honeycombs can be further reduced to

$$\frac{K_I^*}{K_{IC}^*} + \frac{K_{II}^*}{K_{IIC}^*} = 1 \quad (14)$$

3. Foams

Mechanical properties of foams are described well by the bending model of cell walls proposed by Gibson and Ashby [1]. An idealized model of a foam plate with a central crack under a uniform tensile stress is shown in Fig. 4. Using dimensional argument analysis, Maiti *et al.* [8] were able to derive the expression for mode I fracture toughness of foams as a function of

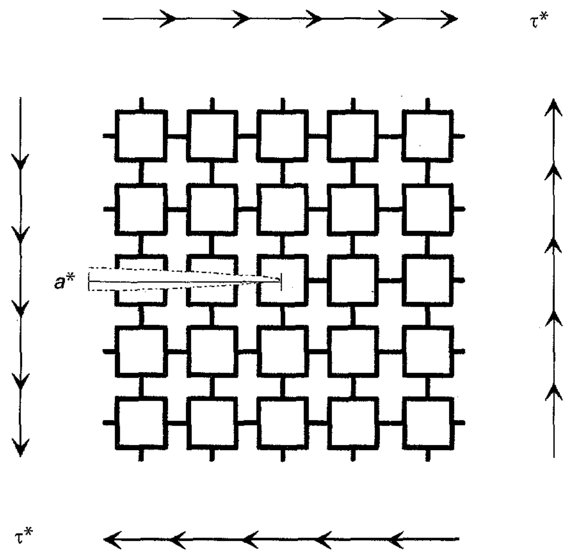

Figure 5 An idealized model of foam plate with a central crack under a uniform in-plane shear stress.

cell size, relative density and the modulus of rupture of solid cell walls

$$K_{IC}^* = C_3\sigma_{fs}(\pi l)^{1/2}\left(\frac{\rho^*}{\rho_s}\right)^{3/2} \quad (15)$$

where C_3 is a microstructure coefficient and was experimentally found to be 0.65 [8].

3.1. Mode II fracture of brittle foams

When an infinite foam plate with a central crack under a uniform in-plane shear stress, as shown in Fig. 5; is of concern, the near-tip singular in-plane shear stress, τ_{xy}, can be utilized again to calculate the total shear force carried by the first unbroken cell wall

$$V \propto \int_0^l \tau_{xy}\, l\mathrm{d}r \propto \tau^* l(a^* l)^{1/2} \quad (16)$$

The critical bending moment exerted at the first unbroken cell wall is simply proportional to the product of total shear force and cell size

$$M \propto Vl \propto \tau^* l^2(a^* l)^{1/2} \quad (17)$$

When the critical bending moment reaches the maximum resistance moment $M_f = \sigma_{fs}t^3/6$ of solid cell walls with a cross-sectional area of t^2, the crack propagates and mode II fracture occurs. Consequently, the fracture in-plane shear strength is

$$\tau_f^* \propto \sigma_{fs}\left(\frac{t}{l}\right)^3\left(\frac{l}{a^*}\right)^{1/2} \quad (18)$$

The relative density of foams is proportional to the square of t/l. The expression for mode II fracture toughness of foams can be obtained once the fracture in-plane shear strength and crack length are known

$$K_{IIC}^* = C_4\sigma_{fs}(\pi l)^{1/2}\left(\frac{\rho^*}{\rho_s}\right)^{3/2} \quad (19)$$

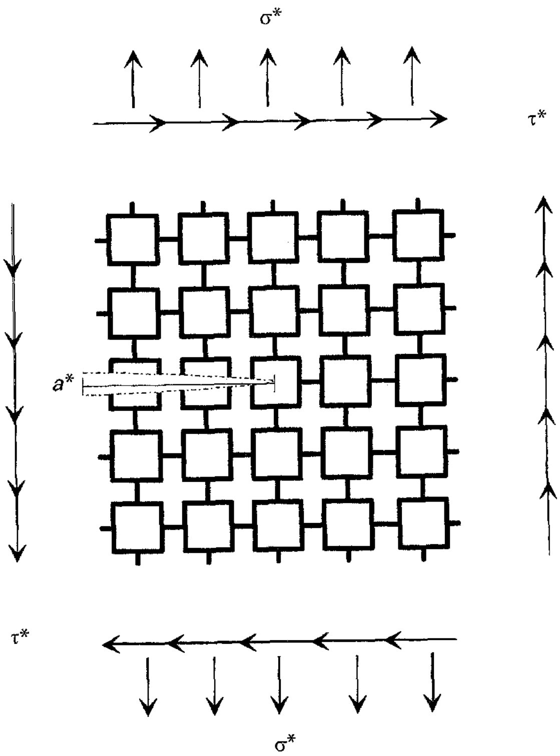

Figure 6 An idealized model of foam plate with a central crack under a combined loading of uniform tensile and in-plane shear stresses.

where C_4 is a microstructure coefficient of foams and must be determined empirically.

3.2. Mixed-mode fracture of brittle foams

When a foam plate is under a combined loading of uniform tensile and in-plane shear stresses as shown in Fig. 6, the bending moment acting at the first unbroken cell wall is a linear combination of the induced bending moments by the uniform tensile stress and by the uniform in-plane shear stress. The critical skin stress of the first unbroken cell wall is found to be

$$\sigma_c \propto \frac{(d_4\sigma^* + d_5\tau^*)l^2(a^* l)^{1/2}}{t^3} \quad (20)$$

where d_4 and d_5 are constants. The critical skin stress increases until it reaches the modulus of rupture of solid cell walls. The maximum combined loading of uniform tensile and in-plane shear stresses $(d_4\sigma^* + d_5\tau^*)_f$, at which crack propagates, is

$$(d_4\sigma^* + d_5\tau^*)_f(\pi a^*)^{1/2} \propto \sigma_{fs}\left(\frac{t}{l}\right)^3(\pi l)^{1/2} \quad (21)$$

The left-hand side of the above expression is again a linear combination of modes I and II stress intensity factors. Thus, the mixed-mode fracture criterion for brittle foams can be expressed as

$$d_4K_I^* + d_5K_{II}^* = d_6\sigma_{fs}(\pi l)^{1/2}\left(\frac{\rho^*}{\rho_s}\right)^{3/2} \quad (22)$$

where d_6 is a microstructure coefficient of foams. The right-hand side of Equation 22, which is dependent on

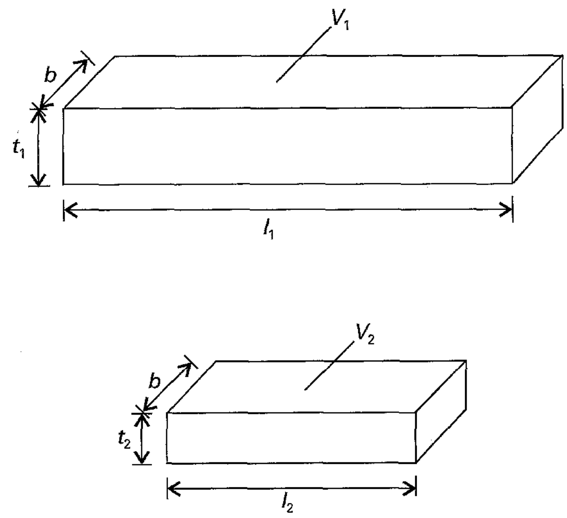

Figure 7 Two solid cell walls for brittle honeycombs with different cell size but the same relative density.

cell size, relative density and material properties, can be converted to mode I fracture toughness and mode II fracture toughness individually

$$\frac{d_4 K_{\mathrm{I}}^*}{K_{\mathrm{IC}}^*/C_3} + \frac{d_5 K_{\mathrm{II}}^*}{K_{\mathrm{IIC}}^*/C_4} = d_6 \tag{23}$$

The above mixed-mode fracture criterion must be valid for the cases $K_{\mathrm{I}}^* = K_{\mathrm{IC}}^*$ for pure mode I and $K_{\mathrm{II}}^* = K_{\mathrm{IIC}}^*$ for pure mode II. Therefore, the mixed-mode fracture criterion for brittle foams can be further reduced to a simple form

$$\frac{K_{\mathrm{I}}^*}{K_{\mathrm{IC}}^*} + \frac{K_{\mathrm{II}}^*}{K_{\mathrm{IIC}}^*} = 1 \tag{24}$$

4. Discussion

K_{IC}^* and K_{IIC}^* of brittle cellular materials increase with relative density and cell size if the modulus of rupture of solid cell walls is regarded as a constant. Also, either K_{IC}^* or K_{IIC}^* is cell geometry dependent. A microstructure coefficient included in each expression of K_{IC}^* and K_{IIC}^* should be determined numerically or experimentally.

The brittleness of solid cell walls, however, will affect their modulus of rupture; brittle solids with a larger volume have a higher value of modulus of rupture. As a result of that, the mode II fracture toughness of brittle cellular materials is controlled by the pre-existing crack-size distribution in solid cell walls. One way to describe the brittleness of solid cell walls is applying Weibull statistic analysis for variable modulus of rupture. Fig. 7 illustrates two solid cell walls for honeycombs with different cell size but the same relative density. That is, $V_1 > V_2$, $t_1 > t_2$ and $l_1 > l_2$ but $t_1/l_1 = t_2/l_2$. The ratio of modulus of rupture for the two solid cell walls can be obtained using Weibull statistic analysis [11]

$$\frac{\sigma_{\mathrm{fs},1}}{\sigma_{\mathrm{fs},2}} = \left(\frac{V_2}{V_1}\right)^{1/m} = \left(\frac{b t_2 l_2}{b t_1 l_1}\right)^{1/m} \tag{25}$$

where m, larger than zero, is the Weibull modulus of solid cell walls. Solids with a lower value of m are more brittle. Because the two solid cell walls have same width, b, and relative density, the ratio of modulus of rupture is further reduced to

$$\frac{\sigma_{\mathrm{fs},1}}{\sigma_{\mathrm{fs},2}} = \left(\frac{l_2}{l_1}\right)^{2/m} \tag{26}$$

It is clear that the modulus of rupture of solid cell walls increases with decreasing cell size for same density honeycombs and the magnitude of increase depends on the Weibull modulus.

The ratio of mode II fracture toughness for two same-density but different cell size honeycombs can be calculated by substituting Equations 26 into Equation 8, gives

$$\frac{K_{\mathrm{IIC},1}^*}{K_{\mathrm{IIC},2}^*} = \frac{\sigma_{\mathrm{fs},1}(l_1)^{1/2}}{\sigma_{\mathrm{fs},2}(l_2)^{1/2}} = \left(\frac{l_1}{l_2}\right)^{\frac{1}{2}-\frac{2}{m}} \tag{27}$$

The above result indicates that there is a cell-size effect on the mode II fracture toughness of brittle honeycombs. When $m > 4$, honeycombs with a larger cell have a higher value of mode II fracture toughness; when $m < 4$, K_{IIC}^* increases with decreasing cell size; when $m = 4$, there is no cell-size effect. A similar result is obtained for brittle foams using the same procedure: K_{IIC}^* of brittle foams increases with increasing cell size if $m > 6$; when $m < 6$, K_{IIC}^* decreases with increasing cell size. The Weibull modulus effect on K_{IIC}^* is the same as that on K_{IC}^* derived by Huang and Gibson [11, 12].

From Equations 14 and 24, it is noted that the mixed-mode fracture criterion for brittle cellular materials is a linear combination of $K_{\mathrm{I}}^*/K_{\mathrm{IC}}^*$ and $K_{\mathrm{II}}^*/K_{\mathrm{IIC}}^*$. For solid materials , the energy-balance criterion requires that the total energy release rate in mixed-mode fracture is the summation of mode I energy release rate and mode II energy release rate [14]. Normally, K_{IC}^* is not equal to K_{IIC}^*. Therefore, a modified mixed-mode fracture criterion is usually applied for solid materials

$$\left(\frac{K_{\mathrm{I}}^*}{K_{\mathrm{IC}}^*}\right)^2 + \left(\frac{K_{\mathrm{II}}^*}{K_{\mathrm{IIC}}^*}\right)^2 = 1 \tag{28}$$

The above equation is a elliptic function of $K_{\mathrm{I}}^*/K_{\mathrm{IC}}^*$ and $K_{\mathrm{II}}^*/K_{\mathrm{IIC}}^*$. That is, the mixed-mode fracture criterion for brittle cellular materials is completely different from that for solid materials.

The mixed-mode fracture criterion for brittle honeycombs and foams must be verified before it is used to examine if a crack under combined loading of uniform tensile and in-plane shear stresses will propagate. However, the experimental results of mixed-mode fracture in brittle cellular materials are limited. Existing experimental results of mixed-mode fracture in PVC foams by Zenkert [7] as shown in Fig. 8 are compared with the theoretical modelling. It is seen that the experimental data of mixed-mode fracture in PVC foams are close to a straight line corresponding

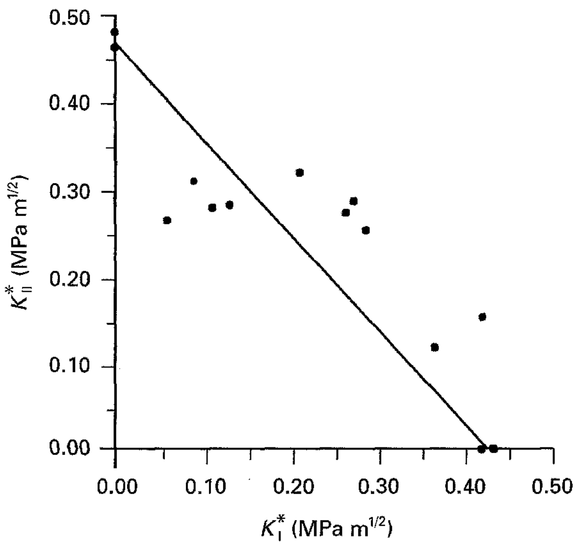

Figure 8 (—) The mixed-mode fracture criterion for brittle cellular materials (Equation 24) compared to (●) experimental results in PVC foams [7].

to Equation 24. Agreement in Fig. 8 supports the view we proposed, giving the confidence of utilizing Equation 24 to check if crack propagation in brittle foams is likely to occur.

At the same time, the mixed-mode fracture criterion for brittle cellular materials can be extended to the case of a combined loading of uniform tensile, in-plane shear and out-of-plane shear stresses. Because solid cell walls are linear elastic up to fracture, the bending moment exerted at the first unbroken cell wall ahead of crack tip is a linear combination of those subject to a single tensile stress, a single in-plane shear stress and a single out-of-plane shear stress. The mixed-mode fracture criterion thus becomes

$$\frac{K_I^*}{K_{IC}^*} + \frac{K_{II}^*}{K_{IIC}^*} + \frac{K_{III}^*}{K_{IIIC}^*} = 1 \qquad (29)$$

where K_{III}^* and K_{IIIC}^* are the mode III stress intensity factor and fracture toughness, respectively.

5. Conclusion

The expression for mode II fracture toughness and the mixed-mode fracture criterion for brittle honeycombs and foams are derived. It is found that both mode I and II fracture toughnesses of brittle cellular materials depend on their cell geometry, relative density and the modulus of rupture of solid cell-wall materials. When the variation of modulus of rupture is taken into account, there is a cell-size effect: K_{IC}^* and K_{IIC}^* increases with increasing cell size for honeycombs with $m > 4$ and for foams with $m > 6$. In addition, the mixed-mode fracture criterion for brittle cellular materials, different from that for solids, is compared with experimental results in PVC foams; agreement is good. Because brittle solid cell walls are linear elastic up to fracture, the mixed-mode fracture criterion is extended to the case of a combined loading of uniform tensile, in-plane shear and out-of-plane shear stresses.

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